Equality versus Progress

John E. Roemer

John E. Roemer *

Equality versus Progress

Consider a society that will last for a finite number, N, of generations. The population size is constant: each individual has one child. At each date, the present generation produces, consumes, and saves for the education of its children. Each adult has a self-interested utility function, whose arguments are consumption and leisure. The problem is to compute the egalitarian tax and investment regime for this society. The wage potential of a child is determined by the amount (of educational resource) that is invested in him, and children from different families are differentially efficient in processing educational investment into future income potential. Thus, each generation (except the last) must decide on a taxation scheme, which has two functions: to raise net investment, to be invested in the education of children, and to redistribute income among the present generation of adults. It must also decide how to invest the educational resource among the children. The last generation has no children, and thus need only decide on a tax scheme to redistribute income. There are thus 2N-1 'instru-

ments' – a tax function for each generation, and a distribution of educational investment among children for N-1 generations. The problem is to choose these instruments to maximize the minimum welfare of all adults who ever exist.

The purest form of this problem occurs when there is no incentive problem: adult labor supply is independent of taxation, and effort of children in school is independent of their future income. In this case, it is easy to see that, at the solution, every person will have exactly the same utility. First, observe that the minimum utility must be equal in all generations. (If the min utility at generation t were higher than the min utility overall, then t should transfer some resources both backwards and forwards.) Then observe that *within* each generation, utility must be maximinned, given the investment it must raise for the next generation; then observe that lump-sum taxes will equalize utility in each generation at the highest possible level.

Thus, the egalitarian solution requires that each person who ever lives be exactly as well

off as the worst-off person in the present ge-
neration. How well off *that* person can be is,
of course, limited by today's technology, even
given that the planner were willing to plunder
natural resources.

When incentives are introduced, the pro-
blem is not so easily solved. Restricting the set
of tax policies to some small set might, per-
haps, help to prevent this solution from
occurring. But I will show, in an example ana-
lyzed in the next section, that even when only
linear income taxation is used, and people
maximize a utility function of the good and
leisure, the same result occurs: to be precise,
that if N is sufficiently large then, at the
maximin plan, almost all adults who ever live
will experience the utility of the worst-off
group of adults in the first generation. The
example in which I show this allows for en-
dogenous technical progress.

Thus, a commitment to egalitarianism
seems to preclude a commitment to 'pro-
gress.' The Left has surely held both com-
mitments to egalitarianism and progress, loo-
king forward to a world in which everyone
has become self-realized, and is, presumably,
much better off than the worst-off person to-
day could be under any realistic tax scheme.
(The restriction to linear income taxes, which
I impose in the next section, is meant to be a
'realistic' restriction.) The worst-off person
today, under any linear income tax scheme,
would not be very well off, yet egalitarianism
commits us to advocating a situation in
which almost everyone, with a suitably long
horizon, will be as poorly off as he. Surely,
this cannot be a situation in which most peo-
ple have become self-realized, a state which
should deliver much more welfare than to-

day's worst-off can possibly enjoy.

My result is a cousin of a well-known result
about intergenerational justice, noted and stu-
died by, among others, Solow (1974), Das-
gupta (1974), and Asheim (1991). Suppose
the problem is to decide on a consumption
and investment plan that maximizes the mini-
mum *average* consumption of all generations,
where investment today will produce goods
available for consumption tomorrow with a
positive rate of return. (There is one good
which serves both for investment and con-
sumption.) Then the maximin plan involves
constant average consumption for all genera-
tions. I find this result less striking than the
one I have described above, for in it there is no
modeling of the intragenerational distribution
of consumption: there is, in effect, one repre-
sentative adult in each generation. In particu-
lar, there is no intragenerational taxation or
modeling of the incentive problem in these
models. The Solow-Dasgupta result does say
that equality precludes progress, but the ver-
dict is not terribly disturbing – if, at any rate,
we think of applying it to a society whose pre-
sent per capita income is quite high. My re-
sult, however, seems deeply disturbing, even
when applied to such a society[2].

An example
Consider an economy that will last for N ge-
nerations; the economy produces one good,
used both for consumption and educational
investment in children. At generation one,
there are two skill levels: fraction f_1 of adults
earn a real wage w_1, and fraction f_2 earn a
wage of w_2, with $w_2 > w_1 > 0$. The utility functi-
on of all adults in all generations is $u(x,L) = x - \frac{\alpha}{2} L^2$, where x is consumption of the good

---

2. There is another concern in the sustainable growth literature that is not of interest to me here, and that is whether
the optimal plan a particular generation computes is time consistent – that is, whether the next generation would
compute the same sub-plan that the present generation has computed for it. (See, for example, Calvo (1978)
and Asheim (1988).) I am only concerned with the true maximin plan that a planner, today, would arrive at.

and L is labor expended. Each adult in every generation except the Nth has one child. Investment in children produces adults at the next period (generation) who have various real wages. If R is the amount of good invested in a child during the educational process, and if the child has a parent whose real wage is w^t, then the child's real wage, when he is an adult at period t+1, is given by

$$w^{t+1} = \delta w^t R^{1/2}. \qquad (1)$$

This 'educational production function' can be interpreted as modeling the fact that the culture of the household is reflected in the parent's real wage, which in turn influences the efficiency with which the child processes educational investment into future productivity. Equation 1 can also be thought of as modeling the process of intergenerational knowledge diffusion: the more the parent knows, the easier it is for the child to learn.

I shall restrict tax schemes to be linear income taxes. Thus, at each generation, a tax scheme $\tau(x) = ax+b$, denoted (a,b), must be chosen. Taxes both redistribute income among present adults, and give rise to net government revenue, which is invested in children, creating adults with skills the next period. Let R be revenue per capita raised in a generation. That revenue must be invested in some way among children; that is, a pair of non-negative numbers $(\varepsilon_1, \varepsilon_2)$ must be chosen, such that

$$f_1\varepsilon_1 + f_2\varepsilon_2 = 1, \qquad (2)$$

and $\varepsilon_1 R$ will be the amount of investment in each child of type i, for i=1,2.

With the given utility function, an adult with real wage w facing a tax scheme (a,b) will offer labor in amount $\hat{L} = (1-a)w/\alpha$; it follows that net government revenue under that tax scheme will be:

$$R = a(1-a)\mu_2/\alpha + b, \qquad (3)$$

where $\mu_2 = f_1 w_1^2 + f_2 w_2^2$ is the second moment of the wage distribution in the generation in question.

Every generation except the last one must raise some non-negative amount of tax revenue for educational investment; the last generation raises zero revenue.

A *society* is specified by the data $(\alpha, \delta, w_1, w_2, f_1, f_2, N)$. A *plan* for the society is a vector

$$\pi = ((a^1, b^1, \varepsilon_1^1, \varepsilon_2^1), (a^2, b^2, \varepsilon_1^2, \varepsilon_2^2), ...,$$
$$(a^{N-1}, b^{N-1}, \varepsilon_1^{N-1}, \varepsilon_2^{N-1}), (a^N, b^N)),$$

which specifies the tax scheme of each generation, and how the net revenue is distributed among children. A plan is feasible if no adult ever is required to pay a tax greater than his income, taking into account the labor supply equation, and if net revenue at generation N is zero.

We may compute that, if an adult has a real wage of w and faces a tax scheme (a,b) which raises a net revenue of R (per capita), then her utility will be

$$v(a,R;w) = (1-a)^2 w^2/(2\alpha) +$$
$$a(1-a)\mu_2/\alpha - R. \qquad (4)$$

(Using the budget balance condition (3), we can eliminate b.) Thus, the multi-generational egalitarian scheme is the one that maximizes the minimum utility of all (potentially) 2N types that live in the society, over all feasible plans.

Let $\rho := (w_2/w_1)^2$. We have:

Theorem
Let $(\alpha, \delta, w_1, w_2, f_1, f_2, N)$ *be a society and suppose* $\rho < 3/2 + f_2/(2f_1)$. *Then, in the egalitarian plan, fraction* $1 - \frac{1-f_1}{N}$ *of all adults who live in the society will experience the utility of the worse off type in generation one.*

The theorem implies that at the egalitarian plan, for long horizons, almost all the adults

who ever live will experience the utility of the worse off type in the first generation.

To prove the theorem, first observe that each generation 1,2,...,N-1 must invest a positive amount in its children, for if a generation of adults failed to do so, then by (1), all adults in all following generations would experience a utility of zero, which is clearly not the maximin solution.

Second, observe that the minimum utility experienced in each generation, at the egalitarian solution, must be the same. This is proved by induction on the number of generations at which the overall minimum utility is experienced. Suppose there were a unique such generation at the optimal plan π. If that generation, call it j, is not the Nth, then we alter the plan by having j invest a little less than it does in π; this increases the min utility at generation j, without reducing the min utility in any other generation too much, and so π was not the egalitarian plan. If that generation is the Nth, we have generation N-1 invest a little more, which raises the utility at generation N. Hence there cannot be a unique generation which experiences the minimum utility. By induction, using similar reasoning, we can show that the number of generations at which the minimum utility is achieved cannot be J, for J = 1,...,N-1. Hence the minimum utility is achieved in every generation.

The third step involves noticing the following. Let π be the optimal plan, which entails a sequence of educational (per capita) investments of $(R^1,R^2,...,R^{N-1})$. Then the distribution of investment $(\varepsilon_1^{t-1},\varepsilon_2^{t-1})$ associated with π in generation t-1, and the tax scheme (a^t,b^t) associated with π at generation t must solve the following program: maximize the minimum utility experienced at generation t subject to the children at t-1 receiving R^{t-1} in education per capita, and subject to generation t's passing on R^t to its children, per capita. This claim is obvious.

The fourth step requires work, which has already been done in another paper, where I show the following:

Proposition A
Let the adults at generation t-1 *have wages* w_1 *and* w_2, *let generation* t-1 *pass on* R^{t-1} *in investment to its children, and let adults at generation* t *pass on* R^t. *Let* $\rho < 3/2+f_2/(2f_1)$. *Then, independent of* R^{t-1} *and* R^t, *the sub-plan* $((\varepsilon_1^{t-1},\varepsilon_2^{t-1}), (a^t,b^t))$ *which maximizes the minimum (adult) utility at generation* t *equalizes the real wages of all adults (at generation* t).

[Indeed, *Proposition A* follows almost immediately from the theorem proved in Roemer (1995, § 3). It's proof is lengthy, and is therefore not reproduced here.]

Now we finish the proof of the theorem. By the premise on ρ and *Proposition A*, it follows that at generation 2, the wages of all adults must be equalized in the egalitarian plan; hence all adults at generation 2 experience the same utility. By step 2 above, that utility must be the same as the utility of the worse off type at generation one. By *Proposition A*, since $\rho = 1$ at generation 2, the wages at generation 3 must be equalized, and so on for all generations. But the utility of all adults in all generations is therefore equal to the utility of the worse off type at generation one, by step 2. Hence all adults in all generations from two on experience the minimum utility, so the fraction of adults in all generations who experience the minimum utility is

$$(f_1 + (N-1))/N = 1 - \frac{1-f_1}{N} \text{ as claimed.} \quad \square$$

Let us make a quick calculation of what the theorem entails. Suppose f_2 =.8 and f_1 = .2 (that is, a minority of 20% is disadvantaged in the society at generation 1). Then the theorem says that as long as $w_2/w_1 < 1.87$, almost all adults in a long-horizon plan experience

utility that the disadvantaged type experiences at generation one. If $w_2/w_1 = 1.8$, for instance, then the disadvantaged type has a real wage only 56% of the real wage of the advantaged type. It can also be shown that the marginal tax rate at the egalitarian plan is never greater than .5. Hence, there will be considerable inequality at the egalitarian plan at generation one, which is to say that the welfare achieved by almost adults in this society will be considerably less than the welfare achieved by the advantaged type at generation one.

If the disadvantaged type comprises only 10% of population, then the bound on w_2/w_1, under which the theorem holds, rises to 2.45.

Finally, it may be worth pointing out that the educational productivity function (1) is not concave: it is characterized by increasing returns. (If one doubles w^t and R, the future adult's wage (w^{t+1}) increases by a factor of 2.82.) So this productivity function should encourage maintaining inequality in wages, because investing a lot in children from advantaged families will produce large increments in output. It is because the possibilities of redistribution through the tax mechanism are quite restricted by its linear form that this increasing-returns effect is overwhelmed by egalitarian considerations, as long as the initial ratio of wages is not too great.

Discussion

Before proceeding, I want to observe that there is a sense in which the chickens come home to roost in the model just discussed. I chose a quite restricted set of possible tax schemes – linear income taxes – to model the idea that, for informational or incentive or political reasons, the planner might not have a great deal of freedom in choosing tax schedules (i.e., she might not have many parameters to set). One thinks, intuitively, that this will keep a substantial fraction of people better off than

they would be were lump-sum taxation possible. But quite the opposite turns out to be case. True, the restriction on the set of tax schemes guarantees that the worst-off group in the first generation will not be very well off – but it turns out that almost everyone will end up being exactly that well off!

Perhaps the most obvious move to take to 'resolve' the conflict between equality and progress is to argue that the utility of adults today should include, as an argument, the consumption or utility of their children, when they become adults next period. Thus, let u(C,L) be the self-interested utility of adults, a function of consumption and leisure, let an adult's *all-encompassing utility*, at generation t, be defined as $v^t(C^t, L^t, v^{t+1})$, where (C^t, L^t) is his own consumption bundle at t, and v^{t+1} is his daughter's all-encompassing utility when she becomes an adult. This move was taken in the sustainable growth literature as well. The usual result here will be that all-encompassing utility, in the maximin solution, will be equalized for almost all adults who ever live, but this *may* be consistent – depending upon the form of v – with ever-increasing bundles of consumption over time (i.e., increasing values of u). This, I think, takes the sting out of the initial result. Although all-encompassing welfare would not increase over time, material well-being, as measured by u, would. This is even consistent with increasing self-realization over time, if we think self-realization is made possible when people escape the jaws of material deprivation. Clearly, however, welfare (as measured by v) would, in that case, not be a measure of self-realization.

Comfortable as this rescue may seem, I will argue that it is unacceptable, for I do not think that the welfare measured by 'v' is the appropriate equalisandum for a theory of justice. Let us think about the rationale for including one's child's consumption as an argument in one's own utility function. That rati-

onale is that one wants one's child to be happy. But in the problem we are considering, the planner who takes 'u' as the equalisandum is in fact computing the plan that will justly distribute resources across generations, in so far as levels of welfare are concerned. I therefore say that including a concern for one's child's level of welfare in the *planner's* equalisandum (i.e., taking 'v' to be the planner's equalisandum instead of 'u') would be double-counting. It may well be that people derive welfare from their children's levels of welfare, but that welfare is not of the sort that the planner, who is concerned with the just allocation of resources across all adults who ever live, should take into account.

Perhaps this point will be strengthened with some examples. Suppose that, perversely, individuals received disutility from their child's happiness: thus, for example, $v^t = u^t - \theta v^{t+1}$. Does it seem ethically attractive, in this case, to use v^t as the planner's equalisandum rather than u^t? Surely not, I think. Well, then why use it when, less perversely, the form is $v^t = u^t + \theta v^{t+1}$? A second example: Suppose that for individual i in generation t the all-encompassing utility is $v_i^t = u_i^t + \theta_i v_i^{t+1}$ – that is, the coefficients θ_i are individualized. Then, the utilities of children will be differentially influenced by how much their parents care about them: again, a repulsive conclusion suggesting that v^t is the wrong equalisandum for the planner.

I see three remaining responses to the result: to argue (1) that although justice requires the derived result, people may be altruistic, and pass down to future generations more than justice requires, (2) that justice does not require intergenerational equality, or (3) that progress is, at the day's end, a retrograde notion, or at least one that is qualitatively less fundamental than equality.

There is a similarity between solution (1) and the view of libertarians like Nozick that,

although justice may produce abject poverty, charity is desirable, to alleviate the severity of justice. But the analogy does not go too far, because intergenerational altruism in our problem will render adults in future generations better off, in terms of functioning (u) than those in the altruistic generation, while Nozick's charitable wealthy ones end up, post-charity, (far, we may assume) better off than their beneficiaries.

In the maximin solution to our problem, generation t passes on resources to the next generation, in the form of education, in amount R^t. Intergenerational charity would involve passing on something more than R^t. If t were the only generation to pass on more than is required by the maximin solution, and the planner were to use that amount to maximize the minimum utility of all adults in all generations after t, then the new solution would entail every post-t adult's utility being greater than the utility of the worst-off adults at generation t. Why should the worst-off adults at t agree to this change? Perhaps, if they are altruistic, they would, but I submit that is a much deeper altruism than Nozick's charitable rich have.

We must ask what the source of the putative altruism is. Why do parents want their children to be better off than they are? Perhaps this is because parents believe progress is important – but the integrity of progress as a value may be impeached: indeed, I shall conclude below that, were we living in the fully egalitarian world of the maximin solution, there would be no rational grounds for valuing progress.

Suppose people in the world our model attempts to portray are indeed altruistic, and do want to pass on more to their children, in the form of educational investment, than justice requires. How should *we*, the scientists of justice, feel about that? I say we should feel sorry about it, which is not to say that the Mi-

nistry of Justice should prevent such charity. We should feel sorry, because in the just solution, everyone is being treated fairly: resources are being distributed in that way which values each human life exactly as much as each other human life is valued – that is, indeed, the universal creed of justice, and hence the creed, in particular, of the egalitarian's kind of justice. If some wish to diminish their own consumption to give others greater consumption than they themselves would then have, we should not prevent them from doing so, but we have no reason to applaud the action.

I conclude that solution (1) is not acceptable. Although people living under the maximin solution might decide to be altruistic and deviate from that solution (although I believe the motivation for such altruism would be far diminished from what we think, as I'll explain below), that should not be a cause for our relief. Egalitarians should, indeed, prefer the maximin solution to its altruistic deviation.

Consider, next, solution (3), or (2) and (3) together, because I intend (2) to be the negation of (3). I think there are three reasons that we value progress: the first is perfectionist, the second is because we think it would be nice if everyone in the future were better off than we are today, and the third is that progress will make the worst-off in the future well off, if not better off than everyone is today.

The perfectionist reason has to do with the perfection of the human species: producing a species of sublime beings, who understand the world about them, and behave cooperatively among themselves. One wonders if valuing this kind of perfection *per se* is, indeed, sensible, for surely one might value the achievement of such a society for its presumed effects on human welfare. But the welfare justification for valuing that beautiful society is not perfectionist. To the degree that the valuation is perfectionist, I oppose it. I value self-realization for what it does for the person who experiences it, not because the perfection of a human being has value in itself.

The second reason, that we value progress because it will make everyone in the future better off than everyone is today, does not stand up to analysis. Those of us whose positive valuation of progress is supported by this reason believe, I propose, that the progress in question can be achieved *without cost* to those living today. We feel that technological progress comes for free, that it simply unfolds with time, without the sacrifice of the present generation for investment in the future. But technological progress is not free: it comes by virtue of investing in the education of children, who, as adults, are then situated to push back the frontiers of their parents' knowledge. *If* we value progress for welfare reasons, and if progress is endogenous and costly to the present generation, then I do not think the second reason stands up to the egalitarian's criticism: there is no good reason to render those in the future better off than those today, at the cost of the latter.

The third reason falls to the same argument that demolishes the second. If we are already in the maximin solution, 'progress' would entail making the contributors to it worse off than its beneficiaries.

Another slant on the welfare arguments for progress, which perhaps helps explain our partiality to it, may be useful. When we value progress – say, in the form of believing that technological progress will entail less material deprivation for all in the future – we are, I think, considering the alternative (to progress) as being that those in the present generation remain in their present condition and that those in future generations do not progress above that condition. But that is not the correct alternative against which to contrast a

world with technological progress. The correct alternative is that the present generation use instead the resources that would have been invested to produce progress (by educating their children to a sufficiently high degree) to ameliorate their own condition. Thus, the fact that there is a *trade-off* between progress and the condition of those who are at present materially deprived is not, I submit, properly taken into account when we think, happily, about progress.

Now in the world that we have, which is not the world of the maximin solution, it is perfectly justifiable for an egalitarian to want progress. For the alternative to progress, given the societies we live in, is not more redistribution to alleviate the material deprivation of those in the current generation; it is, rather, squandering society's actual or potential savings, either in the form of inefficient investment, or increased consumption of those who are already well-off.

I submit that those of us who are egalitarians should not be disturbed by the property of the maximin solution that has been the subject of this discussion. Our intuitions about the value of progress are either perfectionist, or derive from ill consideration of the intertemporal trade-offs necessary for technical progress, or derive from political constraints in our own world which are absent in the maximin solution.

References

Asheim, Geir 1988. "Rawlsian intergenerational justice as a Markov-perfect equilibrium in a resource technology," *Review of Economic Studies* 55:469-484.

---1991. "Unjust intergenerational allocations," *Journal of Economic Theory* 54:350-371.

Calvo, Guillermo. 1978. "Some notes on time inconsistency in Rawls's maximin criterion," *Review of Economic Studies* 45:97-102.

Dasgupta, Partha. 1974. "On some alternative criteria for justice between generations," *Journal of Public Economics* 3, 405-423.

Roemer, John E. 1995. "Equality of opportunity: A theory and examples," Davis: Dept. of Economics, University of California Working Paper.

Solow, Robert. 1974. "Intergenerational equity and exhaustible resources," *Review of Economic Studies*, Symposium, 29-45.